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Albert-Ludwigs-Universität Freiburg
Sommersemester 2024

Topology

Problem Sheet 2

Deadline: 30 April 2024, 15h

Exercise 1 (4 Points)**.**

Let X and Y be topology spaces. Show that a map$f : X \to Y$ is continuous if and only if $f^{-1}\left(\overset{\circ}{A}\right) \subset \overset{\circ}{\left(f^{-1}(A)\right)}$ for all subsets A of Y.

Exercise 2 (4 Points)**.**

Let f and g be continuous maps from topological space X to the Hausdorff topological space Y. Show that f and g are identical if the set

$$\{x \in X \mid f(x) = g(x)\}$$

is dense in X.

Exercise 3 (6 Points)**.**

Let X and Y be topological spaces. A map $f : X \to Y$ has the property $(*)$ if for every sequence $(x_n)_{n\in\mathbb{N}}$ in X converging to x in X, the sequence $(f(x_n))_{n\in\mathbb{N}}$ converges in Y to $f(x)$.

(a) Show that every continuous map $g : X \to Y$ has property $(*)$.

(b) Consider now $X = (\mathbb{R}, \mathcal{T}_{\text{co-count}})$ the real line equipped with the co-countable topology Prove that a sequence $(x_n)_{n\in\mathbb{N}}$ converges in X if and only if the sequence is eventually constant, that is there is some n_0 such that $x_n = x$ for $n \geq n_0$.

(c) Does the converse in (a) hold?

Hint: Identity.

Exercise 4 (6 Points)**.**

Consider the interval $[0, 1]$ as an index set. For every x in $[0, 1]$ let Z_x be $[0, 1]$ equipped with the subspace topology from $\mathbb{R}$ with the euclidean topology. Denote now $Z = \prod_{x\in[0,1]} Z_x$ equipped with the product topology. Every element $(a_x)_{x\in[0,1]}$ in this space can be identified with a function $f : [0, 1] \to [0, 1]$ sending x to a_x.

(a) Show that the collection of injective functions is neither closed nor open in Z.

Hint: Given basic neighborhood B of a function f in Z, which functions belong to B?

(b) Consider now for x in $[0, 1]$ the topological space $Y_x = ([0, 1], \mathcal{T}_{disc})$ with the discrete topology and $Y = \prod_{x\in[0,1]} Y_x$ the corresponding product with the product topology. Determine whether the set of injective funtions is open or closed in Y.

Die Übungsblätter können zu zweit eingereicht werden. Abgabe der Übungsblätter im entsprechenden Fach im Keller des mathematischen Instituts.